Graph Theory

Instructor: Benny Sudakov

Assignment 13

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Let H be an arbitrary fixed graph and prove that the sequence $\frac{ex(n,H)}{\binom{n}{2}}$ is (not necessarily strictly) monotone decreasing in n.

Problem 2: Imitate the proof of Turán's theorem to show that among all the n-vertex K_{r+1}-free graphs, the Turán graph $T_{n,r}$ contains the maximum number of triangles (for any $r, n \geq 1$).

Problem 3: Let X be a set of n points in the plane with no two points of distance greater than 1. Show that there are at most $\frac{n^2}{3}$ pairs of points in X that have distance greater than $\frac{1}{\sqrt{2}}$.

Problem 4: You have 8 batteries and a flashlight that needs two of them to work. Four of the batteries work, the other four don't, but you don't know which ones. You can check two by putting them in the flashlight: it lights up if and only if both of the batteries work. How many tests do you need for the flashlight to light up?