Graph Theory

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Assignment 11

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Let G be a bipartite graph on n vertices. Prove that $\chi_l(G) \leq 1 + \log_2 n$ using the probabilistic method.

Problem 2: Let G be a complete r-partite graph with all parts of size 2. (In other words, G is K_{2r} minus a perfect matching.) Show, using a combination of induction and Hall's theorem, that $\chi_l(G) = r$.

Problem 3: How many spannng trees does $K_{r,s}$ have?

Problem 4: Find the number of spanning trees of $K_n - e$ (the complete graph on n vertices with one edge removed) in two different ways:

(a) using the Matrix Tree Theorem, and

(b) using a double counting argument.