

# Graph Theory

Instructor: Benny Sudakov

## Assignment 10

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

**Problem 1:** If  $G$  arises from a  $k$ -regular graph with  $2m + 1$  vertices by deleting fewer than  $k/2$  edges, then  $\chi'(G) > \Delta(G)$ .

**Problem 2:** Let  $G$  be a connected  $k$ -regular bipartite graph with  $k \geq 2$ . Using that edge-chromatic number of  $G$  is  $k$ , show that  $G$  is 2-connected.

**Problem 3:** Prove that every graph  $G$  of maximum degree  $\Delta$  has an equitable proper  $(\Delta + 1)$ -edge-coloring, i.e. one where each color class contains  $\lfloor e/(\Delta + 1) \rfloor$  or  $\lceil e/(\Delta + 1) \rceil$  edges, where  $e$  is the number of edges in  $G$ .

**Problem 4:** The *Cartesian product*  $H \times G$  of graphs  $H$  and  $G$  is the graph with vertex set  $V(H) \times V(G)$ , with an edge between  $(v, u)$  and  $(v', u')$  if  $v = v'$  and  $u$  is adjacent to  $u'$  in  $G$ , or if  $u = u'$  and  $v$  is adjacent to  $v'$  in  $H$ . Prove that if  $\chi'(H) = \Delta(H) \geq 1$  then  $\chi'(H \times G) = \Delta(H \times G)$ .