Graph Theory

Instructor: Benny Sudakov

Assignment 9

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Let $G_1, G_2, \ldots, G_{100}$ be 100 planar graphs on the same vertex set V, with edge sets $E_1, E_2, \ldots, E_{100}$, respectively, and consider the graph $G = \left(V, \bigcup_{i=1}^{100} E_i\right)$ which is the union of the graphs $G_1, G_2, \ldots, G_{100}$. Prove that $\chi(G) \leq 600$.

Problem 2: For a given natural number n, let G_n be the following graph with $\binom{n}{2}$ vertices and $\binom{n}{3}$ edges: the vertices are the pairs (x, y) of integers with $1 \leq x < y \leq n$, and for each triple (x, y, z) with $1 \leq x < y < z \leq n$, there is an edge joining vertex (x, y) to vertex (y, z). Show that for any natural number k, the graph G_n is triangle-free and has chromatic number $\chi(G_n) > k$ provided $n > 2^k$.

Problem 3:

(a) Prove that $2\sqrt{n} \leq \chi(G) + \chi\left(\overline{G}\right)$ for any graph G.

(b) Let T_1, T_2 and T_3 be three edge-disjoint spanning trees on the same vertex set. Prove that their union is 6-vertex-colourable.