

Graph Theory

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Assignment 4

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Let G be a k -connected graph, where $k \geq 2$. Show that if $|V(G)| \geq 2k$ then G contains a cycle of length at least $2k$.

Problem 2: Let $k \geq 2$ be an integer. Show that if $G = (V, E)$ is a k -vertex-connected graph, then for any k -vertex subset $U \subseteq V$ there is a cycle C in G such that $U \subseteq V(C)$.

Problem 3: Show that if $k > 0$ then the edge set of any connected graph with $2k$ vertices of odd degree can be split into k trails.

Problem 4: Let G be a connected graph that has an Euler tour. Prove or disprove the following statements.

- (a) If G is bipartite then it has an even number of edges.
- (b) If G has an even number of vertices then it has an even number of edges.
- (c) For edges e and f sharing a vertex, G has an Euler tour in which e and f appear consecutively.