# Graph Theory

## Instructor: Benny Sudakov

## Assignment 3

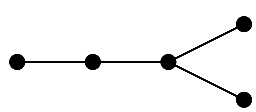
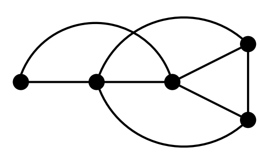
Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

**Problem 1:** Let $G$ be a graph and suppose some two vertices $u, v \in V(G)$ are separated by $X \subseteq V(G) \setminus \{u, v\}$. Show that $X$ is a minimal separating set (i.e. there is no proper subset $Y \subsetneq X$ that separates $u$ and $v$) if and only if every vertex in $X$ has a neighbor in the component of $G - X$ containing $u$ and another in the component containing $v$.

**Problem 2:** Let $k \geq 1$. Show that if $G$ is a graph with $|V(G)| = n \geq k + 1$ and $\delta(G) \geq (n + k - 2)/2$ then $G$ is $k$-connected.

**Problem 3:** Prove that a graph $G$ with at least 3 vertices is 2-connected if and only if for any three vertices $x, y, z$ there is a path from $x$ to $z$ containing $y$.

**Problem 4:** Given a graph $G = (V, E)$, the square of $G$ is the graph $G^2$ obtained from $G$ by adding to it all the edges between vertices at distance 2. For example, if

$G =$  then $G^2 =$  .

(a) Show that if $G$ is connected and $|V(G)| \geq 3$ then $G^2$ is 2-connected.

(b) For every $n \geq 6$, determine $\kappa(G^2)$ in the case where $G$ is a cycle with $n$ vertices.