Graph Theory

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Assignment 1

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Given a graph G with vertex set $V = \{v_1, \dots, v_n\}$ we define the *degree sequence* of G to be the list $d(v_1), \dots, d(v_n)$ of degrees in decreasing order. For each of the following lists, give an example of a graph with such a degree sequence or prove that no such graph exists:

(a) $3, 3, 2, 2, 2, 1$

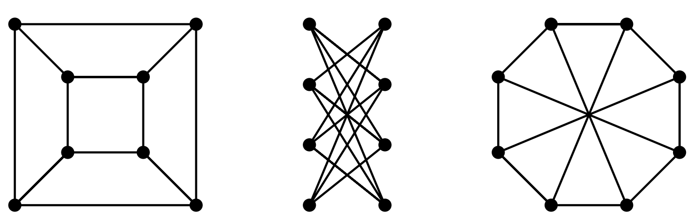
(b) $6, 6, 6, 4, 4, 2, 2$

(c) $6, 6, 6, 6, 5, 4, 2, 1$

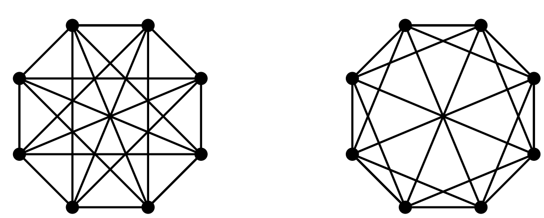
(d) $6, 6, 6, 4, 4, 3, 3$

Problem 2:

(a) Which of the following graphs are isomorphic? Why?



(b) Are the following graphs isomorphic?



Problem 3: Prove that if a graph G is not connected then its complement $\overline{G}$ is connected. Is the converse also true?

Problem 4: Show that every graph on at least two vertices contains two vertices of equal degree.

Problem 5: Prove that every graph with $n \geq 7$ vertices and at least $5n - 14$ edges contains a subgraph with minimum degree at least 6.

Problem 6: Show that in a connected graph any two paths of maximum length share at least one vertex.

Problem 7: Prove that a graph is bipartite iff (if and only if) it contains no cycle of odd length.

Problem 8: Let G be a graph with minimum degree at least 2. Show that there is a connected graph with the same vertex set and the same degree sequence. More precisely, show that there is a connected graph H with $V(H) = V(G)$ such that $\deg_G(v) = \deg_H(v)$ for all $v \in V(G)$.